

# A Formal Model Simulation of Group Learning Behaviors: Group Size and Collaborative Performance

Florin Bocaneala

*Department of Physics, The Ohio State University, 191 W Woodruff Avenue, Columbus, Ohio 43210, USA*

Lei Bao <sup>a)</sup>

*Department of Physics, The Ohio State University, 191 W Woodruff Avenue, Columbus, Ohio 43210, USA*

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Recently, group study has become a common educational strategy employed by educators throughout the country. In this paper, we attempt to construct theoretical models to support and help us understand the reputed efficiency of the group learning over traditional instruction. Specifically, we concentrate on one aspect of the group dynamics: the dynamics of the group emergent semantic agreement.

## I. INTRODUCTION

An extensive literature in psychology and education research analyzes and documents the productivity differences between individual, competitive, and cooperative approaches to classroom instruction and learning (Lazarowitz & Karsenty, 1990; Webb & Farivar, 1994). It is generally accepted that cooperative learning produces greater student achievement than the traditional learning methodologies. According to one of the authors (Slavin, 1995) more than 63% of the cooperative learning groups included in a meta-study have shown an increased productivity as compared with individual learners. In this paper, we construct a formal model of the group dynamics that one may employ to study the mechanisms of the group interaction.

An optimization process of a goal function governs the dynamics of the group: every member of the group apprises the progress made by the group based on an internal goal function. However, the members of the group come to rely on one another in order to satisfy their goals. Consequently, acting within the group places constraints on the process due to communication requirements: successful communication assumes and compels a semantic agreement. Therefore, the interpersonal interaction has to negotiate between individual goal optimization and semantic agreement. One might expect that ontological agreement emerges from this tension.

To investigate the emergent semantic agreement within a group, we designed a system with a variable number of students modeled as finite state machines (Kam, 1997). All the students have access to a common area, a posting board, which they can either read or change their states. This acts as the communication medium that the group members may manipulate (Noble & Cliff, 1996). The finite state machine contains one slot of internal memory state that stores the previous state of the posting board, and one slot that stores one question, which represents the question the student may ask the rest of the group at a time through the posting board. In this way, any action the student might take when its turn comes is determined by the present state of the posting board and the known question. No student may share the content of the internal question slot directly with any other

student: they can communicate only through the posting board, using the limited set of symbols that makes up their vocabulary. However, the way in which each group member uses this vocabulary is dependent on the semantic conventions specified by its own behavioral matrix. This situation reproduces the real case in which a student doesn't know what another student is thinking; he just receives an encoded message in a sound sequence.

This paper is not concerned with the learning per se, or to put it differently, it regards learning from a different perspective. The finite state machines are quite rudimentary and lack the nuances that human cognition entails. Our choice was motivated by the assumption that learning could be modeled as a language acquisition process (from some point of view, learning physics or any science could be even more problematic, because there are no decent dictionaries: one learns through an elaborated process of corroborating behavior and discourse; see for example (Kuhn, 1996)) and in as far as the acquisition process is concerned, even a rudimentary one would work. The emphasis of this simulation falls on communication and how it emerges along with semantic agreement—and learning itself — during the group performance of an educational task.

The language employed during this simulation was built around a limited vocabulary of symbols that group members could place on the common posting area. Generally speaking, real languages involve more complicated phenomena than what may be reproduced using such a limited vocabulary. However, technical jargon involves a limited number of terms. Moreover, a group of students working within a learning task usually experiments and explores the given problem, struggling to describe it using a finite set of words.

During their explorations, utterances may be tentative, searching, and often fragmented because students are uncertain of the technical language usage, although they may have expectations (regarding the actions generated by their words since most of the time ordinary vocabulary does over-

lap with the technical vocabulary) based on responses to previous discourses, and memories of previous experiences. This remembrance of past experiences is part of each learner's personal knowledge. Exploratory talk is an acceptable, even necessary, way for speakers to bring in their tacit knowledge, or "personal knowledge" (Polanyi, 1958), such that through socialization they construct new meanings.

## II. FINITE STATE MACHINES

Finite State Machines (FSM) are models of behavior for a system or a complex object. The essential features exhibited by these behavioral models are that they can model systems encompassing a limited number of conditions or modes, and that they are built with a specified way of learning or changing their behavior in response to changes environmental circumstances. Finite state machines are typically used as a type of control system in which knowledge is represented by states, and actions are constrained by rules. They are extremely simplified rule-based behavioral systems, which is exactly their strength. One may employ them to code behavior in a way that is not excessively demanding of resources.

There are four main elements that go into the design and building of a finite state machine:

1. A finite state machine encodes behavioral states that may generate actions, provided that a certain set of conditions are met.
2. There is a clearly specified way for the finite state machine to change its internal state, switching from one to another.
3. A finite state machine contains rules or conditions that must be met to allow a state transition.
4. A finite state machine accepts input events that are either externally or internally generated, which may possibly trigger rules and lead to state transitions.

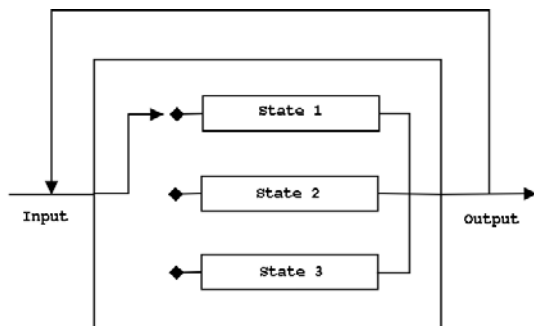


Figure 1. A finite state machine flow chart

One may visualize a finite state machine as a matrix having the dimension of the input space, keeping in mind

that the input space contains all possible events generated from outside or inside the finite state machine. The simulation must start from an initial state. During the simulation this state will subsequently be modified by the interaction with the input events, but the finite state machine must be able to remember the product of the last state transition. In particular, received input events act as triggers that cause an evaluation of the rules that govern the transitions from the current state to other states. In Figure 1, we represent the flow chart of a finite state machine run with three internal states.

## III. SEMANTIC AGREEMENT AND GROUP SIZE

We return now to our simulation. Before the simulation starts, we initiate the FSM behavioral matrices by use of random values in order that the group members begin with different semantic rules: none of them knows the semantic rules used by its neighbors when posting on the common board. Then, we allocate random questions from the training environment to the students to store in their internal states. The training environment of this simulation is made up from a set of symbols, each symbol corresponding to one question a student may ask or be asked with respect to a given task or problem.

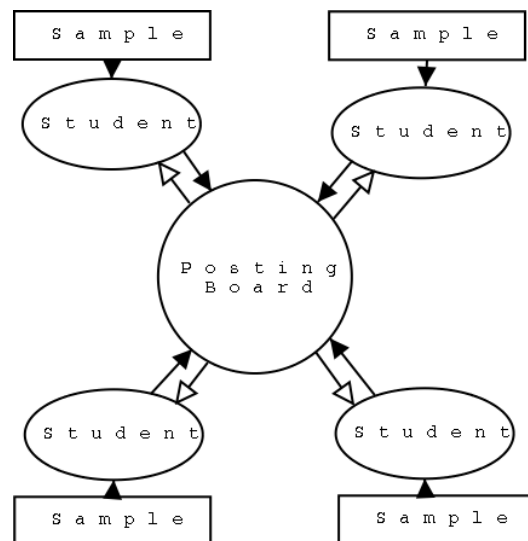


Figure 2. Finite state machines around a common posting board

The simulation starts by sweeping the entire group a sufficient number of times (this number depends on the size of the vocabulary used and was chosen such that the denotational matrix — see below — has enough time to reach a stable region), prompting each FSM to either post its internal question as a question (symbol) for everyone else to see, or try to answer to the posted question (symbol). If the student who posted the question finds the answer incorrect, the student who answered updates the behavioral matrix rule employed accordingly in order to increase the chance of

answering appropriately next time. Thus, the student who attempted to answer is able to learn. If the answer is, however, found appropriate, the student who answered receives a reward. In this way the simulation keeps track of the students' performance.

After these sweeps of the group, the students receive fresh questions from the training environment and the process is repeated. When the group members have renewed their internal questions another number of times, the simulation decides which group member has the least amount of rewards, in other words, the student who consistently did not agree with the rest of the group. This student will simply discard its behavioral matrix and make up a new one composed from the two members of the group who performed consistently the best:

$$c_{ij} = \theta(in + j - r_1)\theta(r_2 - in - j)b_{ij} + [\theta(r_1 - in - j) + \theta(in + j - r_2)]a_{ij},$$

where  $a_{ij}$  and  $b_{ij}$  are elements of the behavioral matrices corresponding to the best and next best group members (for simplicity, we have given a formula for square matrices of size  $n$ ; the actual simulation used a generalization of this formula to three-dimensional matrices),  $\theta$  is the step function, and  $r_1$  and  $r_2$  are two random integer numbers  $r_1 < r_2$ . The reconstruction formula makes sure that the reconstructed behavioral matrix preserves the symbol-action relations (semantic rules) that the parent behavioral matrices are using. Also, the reconstruction formula is general enough to assure a combination between the semantic rules of the parent matrices without becoming implausible: the reconstructed matrix inherits one compact subset of semantic rules from one parent and one compact subset from the other.

On top of this combination between the best two behavioral matrices, there is a certain probability that some elements of the matrix are compromised during copying. In this way, the simulation makes the assumption that in real life, when one of the group members reconstructs the semantic rules or the understanding of a problem domain, as long as the group is well-balanced in terms of expertise, the worst student will employ mainly the two best choices. We also need to mention that this is the only point in the algorithm at which we introduce a scale. This fact will become important in the following sections.

Forty iterations of the sort described above make up a training epoch.

During each epoch we compute the denotational matrix (Hermanns & Rettelbach, 1994). This matrix records all the communication events associating a given answer with an expected answer: each time one group member gives an answer for the posted question, the element of the denotational matrix corresponding to the given answer and the answer that the student who posted the question was expecting is incremented. Therefore, the denotational matrix is a correlation matrix between expected and provided answers.

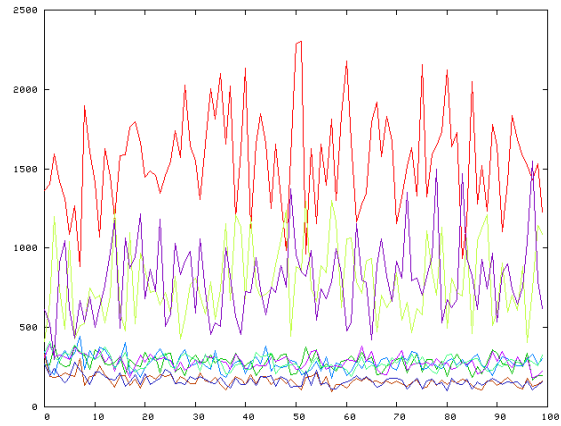


Figure 3. The elements of denotational matrix for a 4 student group vs. epoch number

Initially, this matrix is uniform because we initialized all the FSM behavioral matrices randomly: in the beginning there is basically no correlation between expected and attempted answers because the group members employ arbitrary semantic rules. As the students begin to understand one another, the denotational matrix becomes singular. As we see in Figure 3, three elements of the denotational matrix separate from the rest, and, after some number of epochs, they reach a plateau where the group of students has a semantic agreement. In order to evaluate the singularity of the denotational matrix, once the matrix reaches its plateau, we start by identifying the peak elements in the matrix, and then reorganizing the columns and the lines of the matrix such that the peak elements line up on the diagonal. The average distance between the peak diagonal elements and the off-diagonal elements will characterize the singularity of the matrix:

$$\sigma = \frac{1}{2(n-1)n} \sum_{i=1}^n \sum_{j \neq i} 2m_{ii} - m_{ij} - m_{ji},$$

where  $m_{ij}$  is the denotational matrix,  $\sigma$  is its singularity, and  $n$  the dimension of the matrix or the cardinality of the vocabulary set.

One observes a peak in this average distance for the four-student group in Figure 4, result which agrees with experimental results: starting from data describing academic achievement and active engagement in small study group, J. D. Hagman and J. F. Hayes recommend group sizes of four-five students (Hagman & Hayes, 1984; see also Johnson & Johnson, 1989). In general, the psychology and education literature agrees that groups of four or five members work best. Larger groups decrease each member's opportunity to participate actively and have a negative impact on the academic performance, especially in the case of lower skill students. Other factors also contribute to the optimum size of the study groups. The amount of time available for instruction seems to relate proportionally with the group size: less available time recommends smaller group sizes such

that each group member receives a larger amount of individual instruction time. The skill distribution of the students making up the group goes also proportionally with the group size: less skillful students need more instruction time, speculate the authors (see, for example, Cooper, 1990; Johnson et al., 1991; and Smith, 1986).

Two large-scale studies investigating the impact of instructional group size and academic achievement drew essentially the same conclusion: Larger group sizes correlated inversely with academic achievement for special education students. Gottlieb and Alter (1997) based this conclusion on their evaluation of mandated increases from five to eight students in New York City resource and speech language classrooms. Results from statewide reading achievement tests revealed that only 16% of sixth graders met state reading criteria after group size increases, compared with 29% before increases (1994-95).

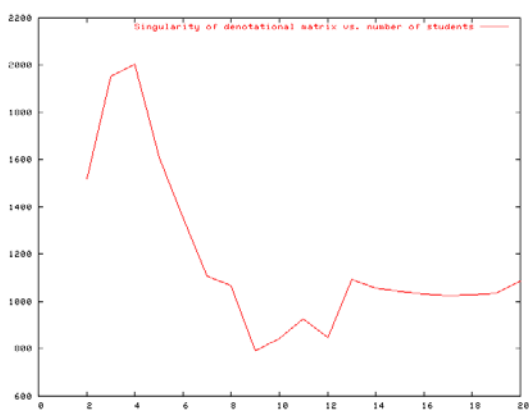


Figure 4. The singularity of denotational matrix vs. number of students

Summing up the phenomenological results, there are two main factors that seem to influence the relation between group size and academic performance. First, the instruction time share for each individual member of the group: more students in a group decrease the amount of individual instruction time per group member. Following this line of argumentation one has to favor smaller groups over bigger groups, and even expect that single student groups should be the optimum instructional formula. However, the experimental findings show that larger groups (4-5 student groups) have a more beneficial impact on group and individual performance.

Second, the ability of the group to emulate and support active engagement of its members has a better chance to explain the phenomenological results. On one hand, a single member group does not engage the student much because there is no peer to support and entice the student. On the other hand, being part of a large group encourages isolationism because the group members wouldn't have many opportunities to contribute actively to the group discussions. Our simulation, starting from different assumptions and monitoring a factor different from the two phenomenological factors

discussed above, corroborates the experimental results and proposes a different way to look at them.

The peak in Figure 4 has been obtained starting from the assumption that one may judge the group performance based on the quality of the semantic agreement the group reaches. There are two essential ingredients responsible for this result. In the first place, one has to count the ability of the group members to learn and adjust their behavioral rules based on the group interaction. Although necessary, this feature of the simulation is not sufficient: the denotational matrix does not become singular if one turns off the reconstruction step (the singularity of the denotational matrix remains to lower values). Therefore, one has to extend the simulation to include the other essential ingredient: the reconstruction step.

It turns out that the reconstruction step in the simulation is essential for two reasons. First, the denotational matrix does not become singular without this step even after very long simulation times. Apparently, the simple learning and local adjustment of the behavioral matrix is not enough for reaching a semantic agreement. From this point of view, the "worst" students in the group initiate the semantic compromise necessary for a subsequent agreement.

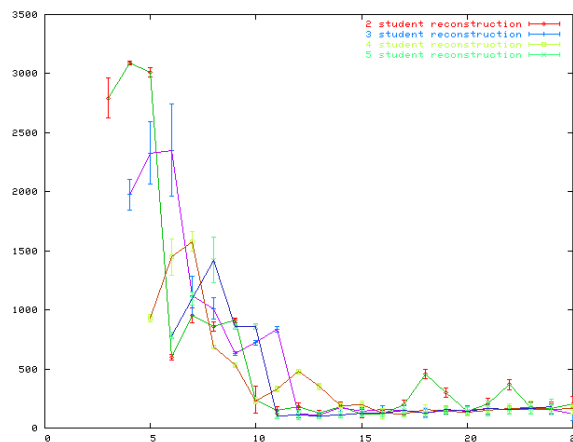


Figure 5. The singularity of denotational matrix vs. number of students for different reconstruction algorithms

Second, the reconstruction step introduces a scale in the simulation that relates to the position of the peak in Figure 4. In order to see this, we generalized the reconstruction algorithm to involve more than two students and plotted the singularity of the denotational matrix versus the group size for scenarios when 2, 3, 4 and 5 group members contribute to the reconstruction of the "worst" student. Figure 5 shows that the peak in figure 4 does shift with the scale change. We see this finding as an experimental challenge: one may go and set up an experiment analyzing relatively homogeneous study groups of varying sizes for the optimal number of group members that seem to influence the group outlook in a larger extent.

#### IV. SEMANTIC AGREEMENT AND CROSS GROUP COMMUNICATION

The next question we asked was, What will happen if we extend the above simulation to more groups? How will communication across groups influence the previous results? Therefore, we have settled for two groups of four but we extended the options a “student” has by giving the student an opportunity to ask, or listen, to a “teacher” in an adjacent group. Specifically, this time our “classroom” contains two four student groups and each group member may, when its turn comes, post a question, answer a posted question, or learn (with a certain standard probability) the answer to the posted question from the best student in the adjacent group. This latter learning was implemented mechanically, the student simply updating the internal state in the FSM matrix with the corresponding state of the “teacher”’s.

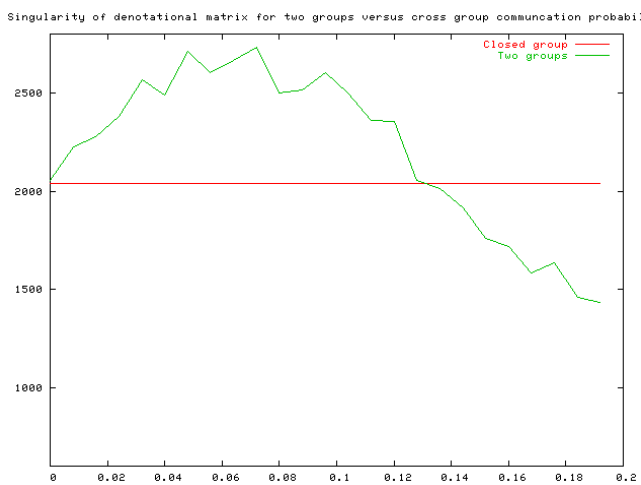


Figure 6. The singularity of denotational matrix versus cross-group communication probability

For relatively large probability the singularity becomes smaller, which was expected given that asking students from the adjacent group is equivalent with extending the numbers of group members, but as we have seen already the singularity in that case peaks at around four group members. What is interesting, though, is the fact that for relatively small probabilities the singularity for this simulation is superior to a close group singularity for the same group size. Therefore, the simulation seems to suggest that cross-group communication is beneficial to reaching a semantic agreement as long as the amount of extra group communication is kept to lower values. This result corroborates experimental results (Hagman & Hayes, 1984; Johnson & Johnson, 1989). Also, Robert J. Beichner (Beichner, 2000; Handelsman et al., 2004) has implemented a modified version of PBI tutorials developed by the PER group at the University of Washington: instead of organizing students in separate work groups, he placed three groups of three students each around a 6-foot round table in order to encourage cross-group interaction. However, every three student group was evaluated as an independent unit and students were encouraged to collaborate mainly within their group; interaction with the stu-

dents in the adjacent groups was accepted only when the entire group reached an impasse, and before the group would request the assistance of an instructor. He reports significantly improved performance in problem solving and increased conceptual understanding: 43% and 50% average normalized gain on FCI as compared with only 23% normalized gain for traditional classes.

#### V. CONCLUSIONS

The internal dynamics of the study group is determined by the quality of the communication. Moreover, in a certain extent, one may see a learning process taking place within a group as similar to a process of reaching a semantic consensus in that group. This paper proposes a scenario for a group semantic play that seems to conduce to the emergence of the semantic agreement within the group. As a side result of the simulation, making the assumption that two of the players of the semantical game influence the rest of the group, the simulations suggest that a group size of four is the optimum size for reaching the highest semantic agreement. This argument, seen in reverse, suggests an experimental test for the above theoretical assumption.

#### ENDNOTES AND REFERENCES:

- a) Email: bao.15@osu.edu, to whom correspondence concerning this article should be addressed.
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